Imaging Interiors: An Implicit Solution to Electromagnetic Inverse Scattering Problems

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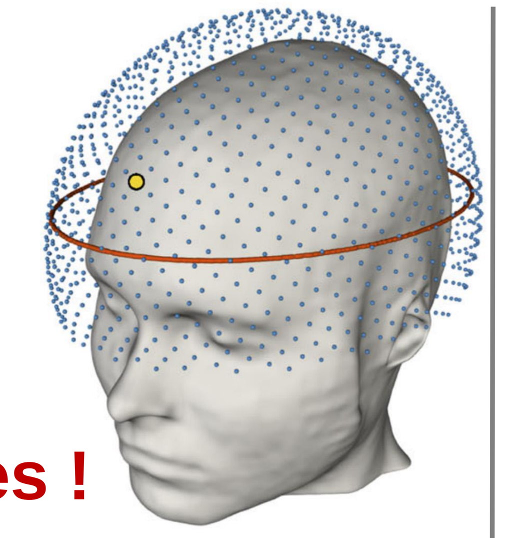
Project Page

WeChat

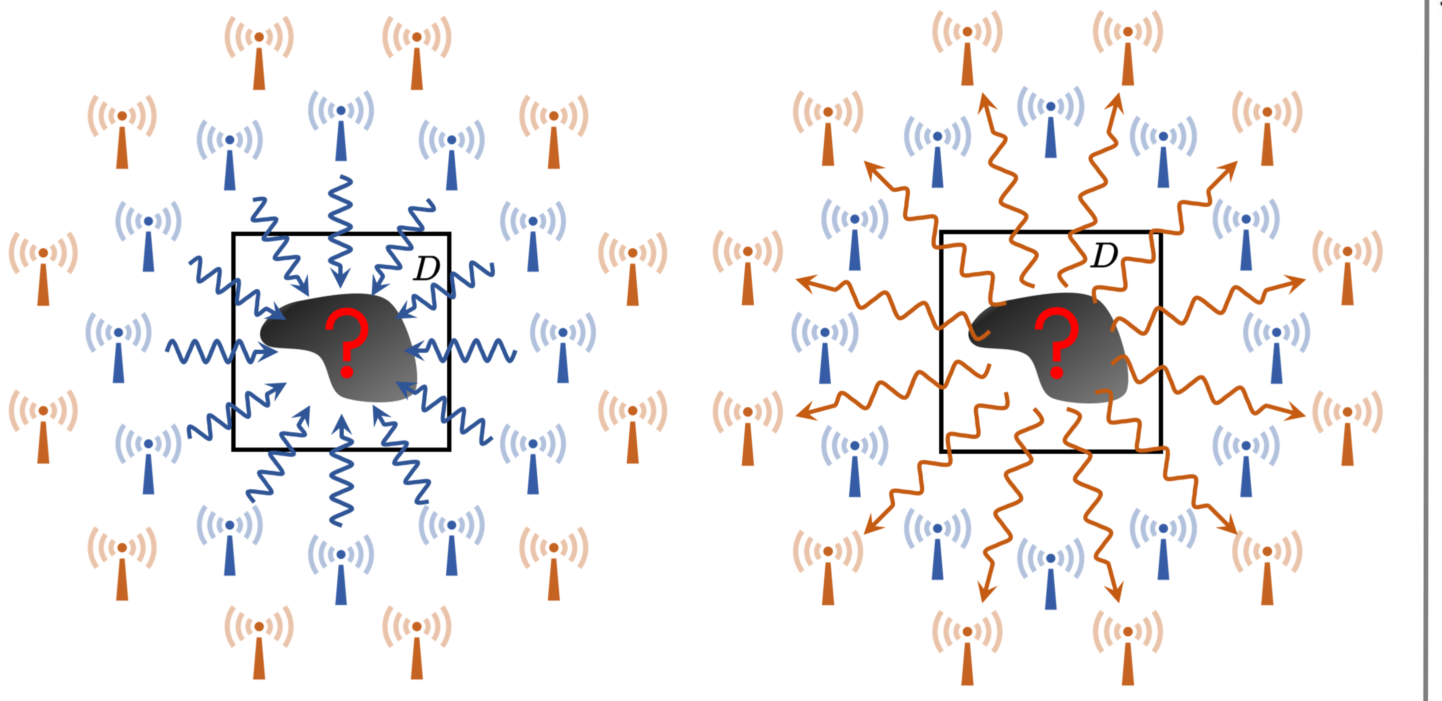
How can we non-invasively visualize the interior of the human body?



Using electromagnetic waves !

Problem Statement

➢ **Setup**



① Transmitters send electro-magnetic waves to the object

② Receivers receive scattered waves from the object

Goal: Reconstruct the object's properties (relative permittivity) from the measured scattered waves.

➢ **Formulations**

- State equation:

$$\mathbf{E}^{\mathrm{t}} = \mathbf{E}^{\mathrm{i}} + \mathbf{G}_D \cdot \mathbf{J}$$

- Data equation:

$$\mathbf{E}^{\mathrm{s}} = \mathbf{G}_S \cdot \mathbf{J}$$

- Relationship between $\mathbf{J}$ and $\mathbf{E}^{\mathrm{t}}$:

$$\mathbf{J} = \mathrm{Diag}(\boldsymbol{\xi}) \cdot \mathbf{E}^{\mathrm{t}}$$

$$\boldsymbol{\xi} = \varepsilon_r - 1$$

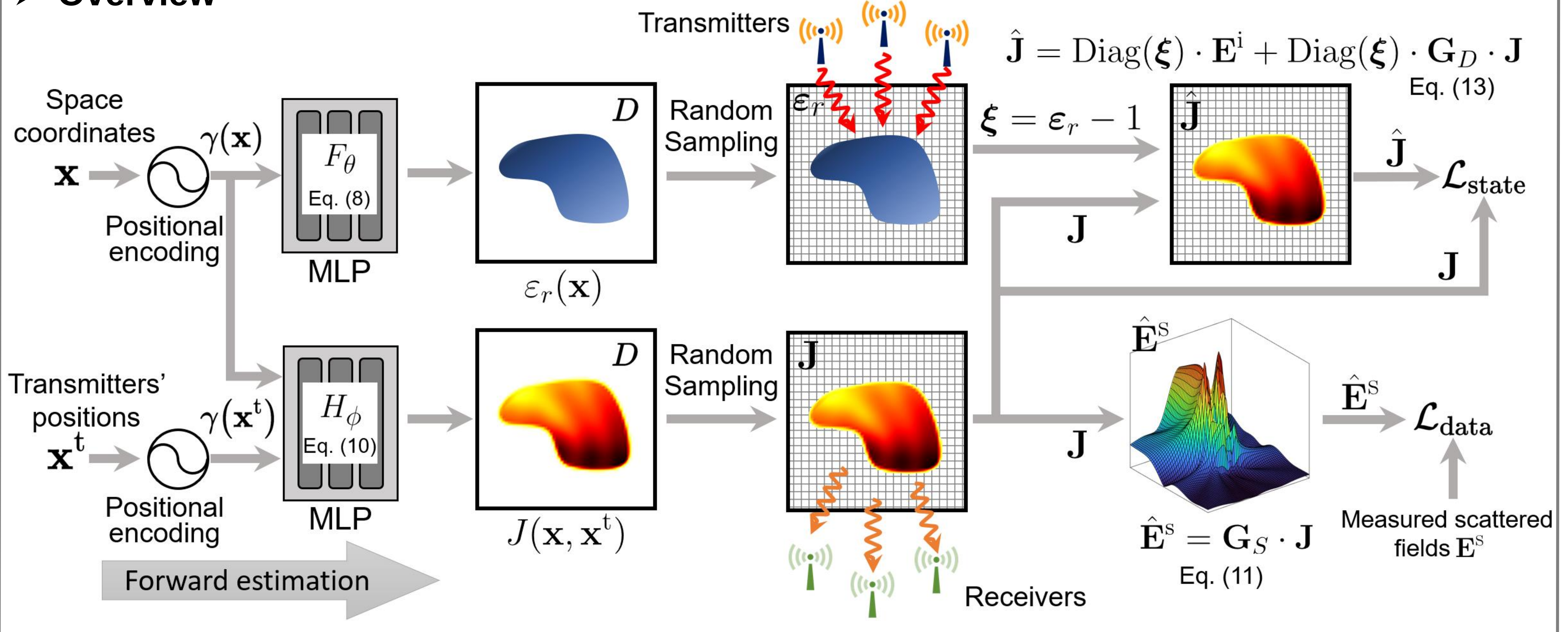
$\mathbf{E}^{\mathrm{t}}$ Total fields
$\mathbf{E}^{\mathrm{i}}$ Incident fields
$\mathbf{E}^{\mathrm{s}}$ Scattered fields
$\mathbf{J}$ Induced current
$\mathbf{G}_D$ Green's function
$\mathbf{G}_S$ Green's function
ε_r Relative permittivity

Known: $\mathbf{E}^{\mathrm{i}}, \mathbf{E}^{\mathrm{s}}, \mathbf{G}_D, \mathbf{G}_S$

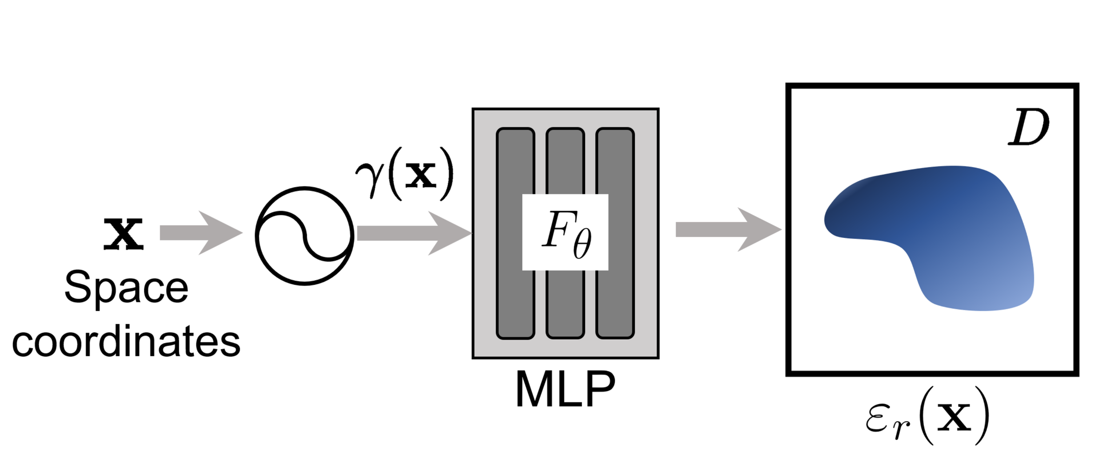
Reconstruct: ε_r

Method

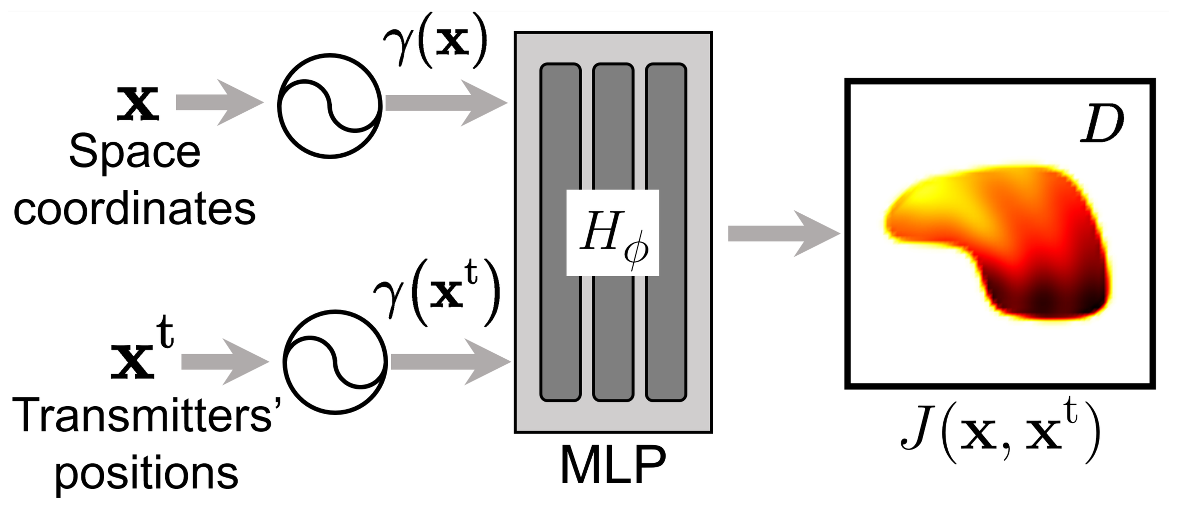
➢ **Overview**



➢ **Representation for relative permittivity**



➢ **Representation for induced current**



$$\varepsilon_r(\mathbf{x}) = F_\theta\left(\gamma(\mathbf{x})\right) \qquad J(\mathbf{x}, \mathbf{x}^{\mathrm{t}}) = H_\phi\left(\gamma(\mathbf{x}), \gamma(\mathbf{x}^{\mathrm{t}})\right)$$

$$\gamma(\mathbf{x}) = \left[\sin \mathbf{x}, \cos \mathbf{x}, \cdots, \sin 2^{\Omega-1}\mathbf{x}, \cos 2^{\Omega-1}\mathbf{x}\right]^\top$$

➢ **Forward calculation based optimization**

- **Data loss**:

$$\hat{\mathbf{E}}^{\mathrm{s}}_p = \mathbf{G}_S \cdot \mathbf{J}_p$$

$$\mathcal{L}_{\mathrm{data}} = \sum_{p=1}^{N_{\mathrm{t}}} \|\hat{\mathbf{E}}^{\mathrm{s}}_p - \mathbf{E}^{\mathrm{s}}_p\|^2$$

- **State loss**:

$$\hat{\mathbf{J}}_p = \mathrm{Diag}(\boldsymbol{\xi}) \cdot \mathbf{E}^{\mathrm{i}}_p + \mathrm{Diag}(\boldsymbol{\xi}) \cdot \mathbf{G}_D \cdot \mathbf{J}_p$$

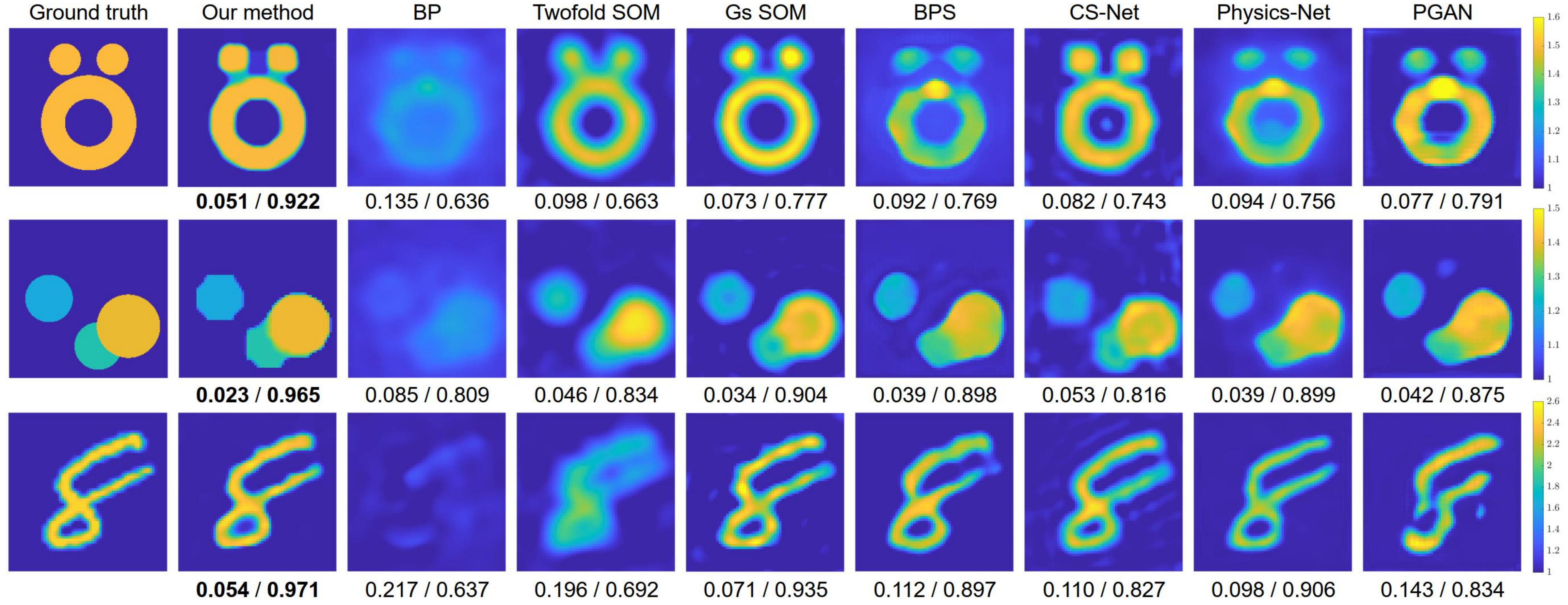
$$\mathcal{L}_{\mathrm{state}} = \sum_{p=1}^{N_{\mathrm{t}}} \|\hat{\mathbf{J}}_p - \mathbf{J}_p\|^2$$

- **Overall loss**:

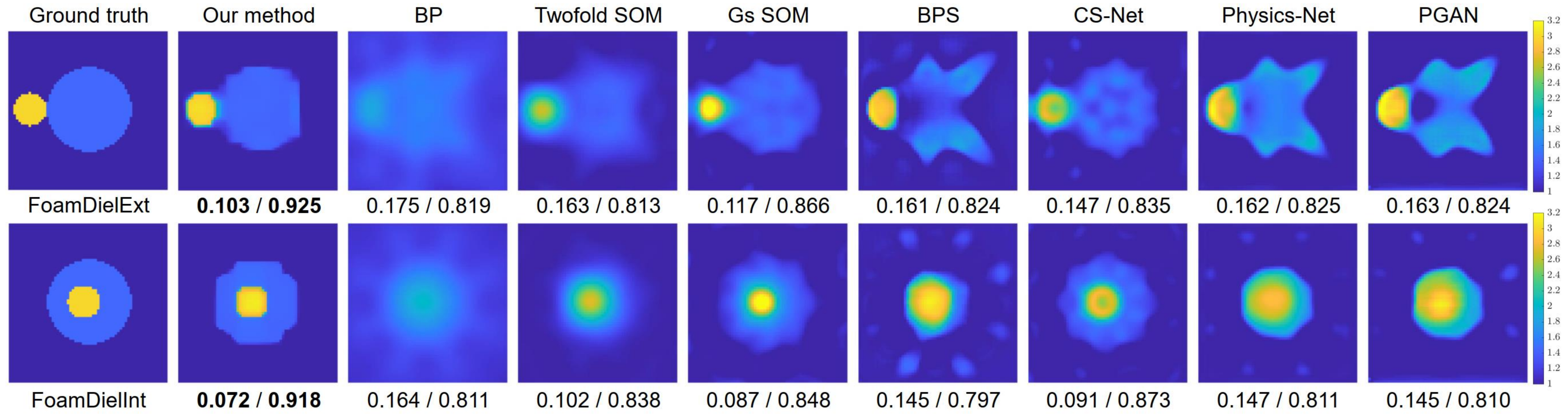
$$\mathcal{L} = \lambda_{\mathrm{data}}\mathcal{L}_{\mathrm{data}} + \lambda_{\mathrm{state}}\mathcal{L}_{\mathrm{state}} + \lambda_{\mathrm{TV}}\mathcal{L}_{\mathrm{TV}}$$

Results

➢ **Results on Synthetic dataset**



➢ **Results on Real-world dataset**



➢ **Results on 3D dataset**